Sensitivity Analysis of a Spatio-Temporal Hydrological Model for Pesticide Transfers

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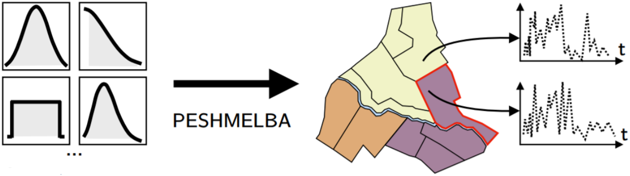
Claire Lauvernet, Arthur Vidard

Katarina Radišić

30 November 2021

INRAE Lyon currently developing PESHMELBA model *[Rouzies et al. 2019]*.



Simulates water and pesticide transfers on a watershed scale, while considering the heterogeneity of landscape elements (plots).

Type of output studied : surface moisture outputs.

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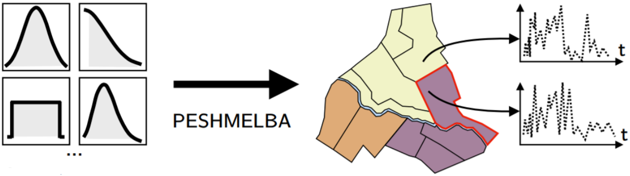

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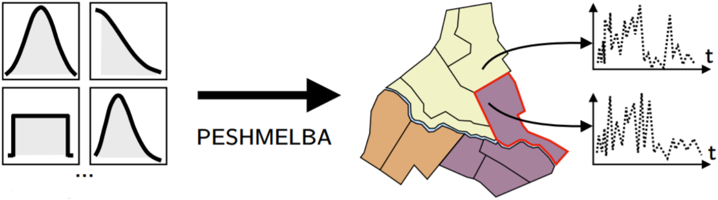
Perform a **sensitivity analysis** on PESHMELBA outputs while taking into consideration both the **temporal** and the **spatial** aspect.

Introduction

Challenges on PESHMELBA



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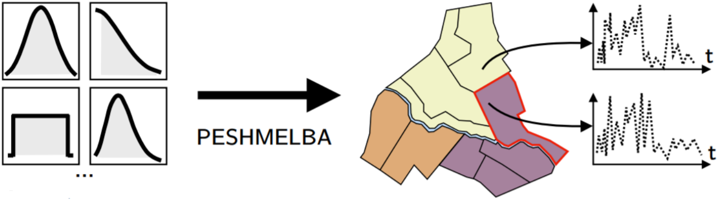
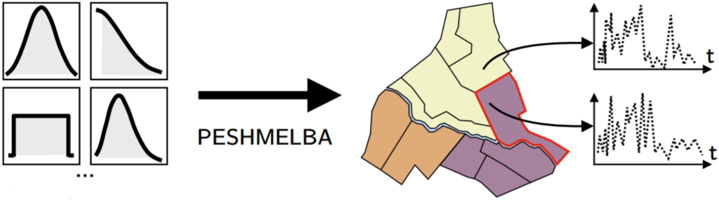


$$\begin{bmatrix} Y^{(1)}(t) \\ Y^{(2)}(t) \\ \vdots \\ Y^{(M)}(t) \end{bmatrix} = \mathcal{M}(\boldsymbol{X}, t), \quad t \in \mathcal{T}$$

where M is the number of areal units.



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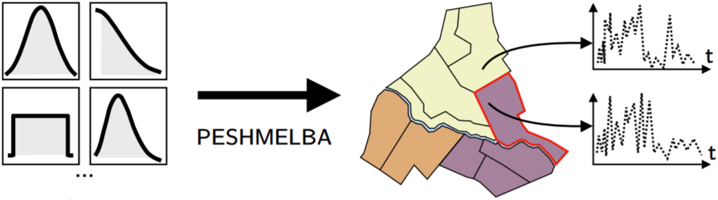

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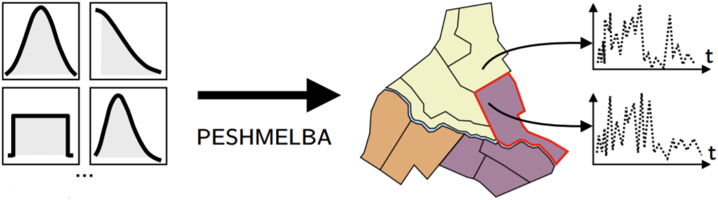
- spatial interactions



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- time dependent outputs
- spatial interactions
- high number of input parameters (145)

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Morris method : computationally cheap, classifies input parameters in two groups, **influential and non-influential**.

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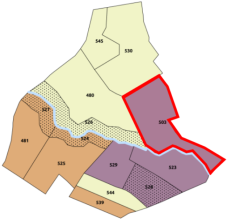

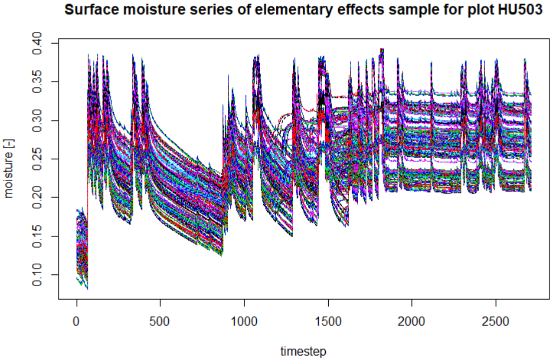

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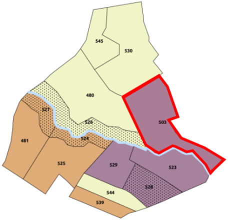
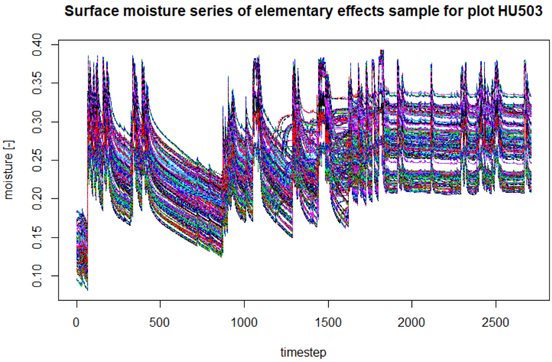
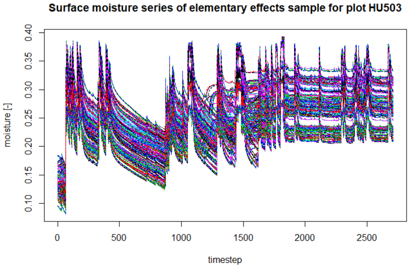
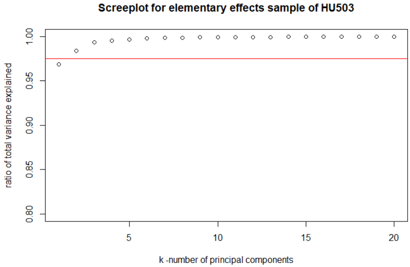


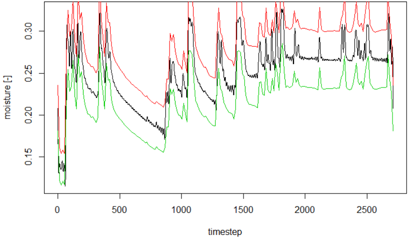


Apply the Morris method to the scores on the **functional principal components** of one areal unit at a time.

1. Screening


Surface moisture series of elementary effects sample for plot HU503
moisture [-]
timestep


Screeplot for elementary effects sample of HU503
ratio of total variance explained
k -number of principal components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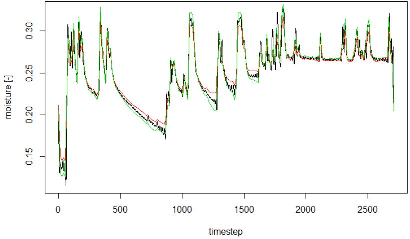
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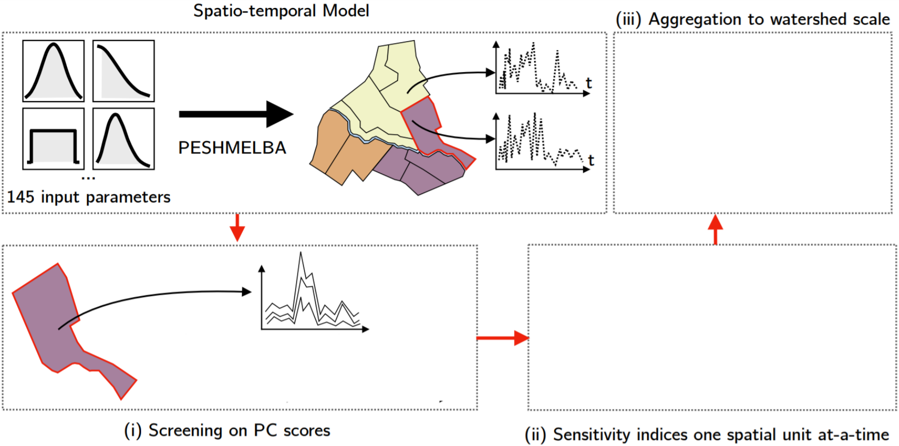
Spatio-temporal Model
145 input parameters
PESHMELBA
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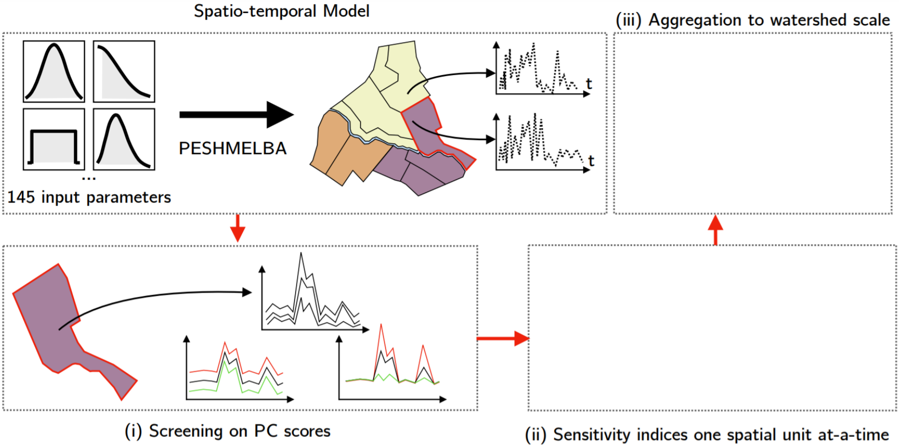
(iii) Aggregation to watershed scale
(i) Screening on PC scores
(ii) Sensitivity indices one spatial unit at-a-time



Input parameters reduced from 145 to 52 influential parameters at the watershed scale.

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The output is scalar now, we fall back into the classical formulation for Sobol' indices calculation.

Polynomial Chaos Expansion (PCE) metamodel :

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Sobol' indices are obtained analytically from PCE :

$$S_i = \sum_{\alpha \in \mathcal{I}_i} y_\alpha^2 / D$$

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$$Y = \sum_{\alpha \in \mathcal{A}_q^{K,p}} y_\alpha \Psi_\alpha(\boldsymbol{X}) + \epsilon_{truncation}$$

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The precision of the Sobol' indices obtained depends on the precision of the PCE metamodel w.r.t. the real model $\mathcal{M}$.

3. Temporal aspect

3.3. Results for Sobol' indices on one areal unit at a time

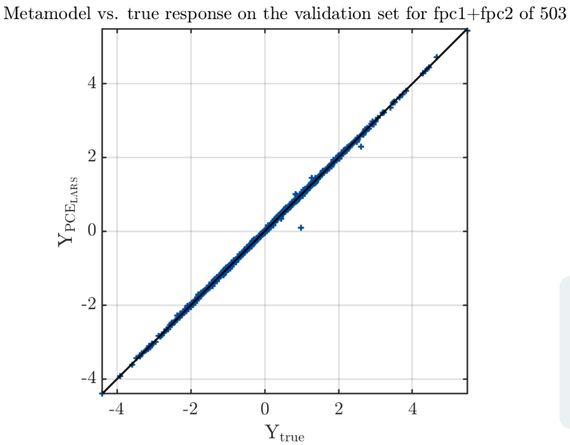
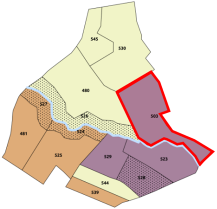
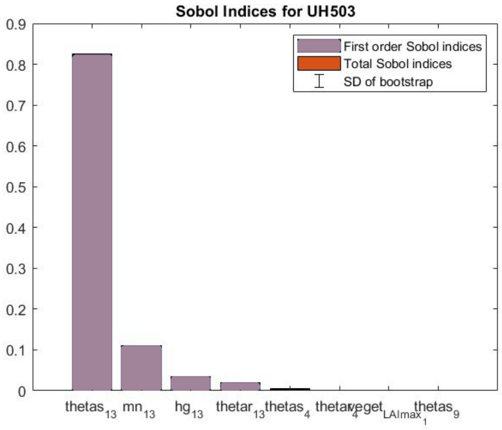


Figure – Validation set metamodel output vs PESHMELBA simulation.



Quality of the PCE metamodel : $R^2 > 0.95$ on validation set.



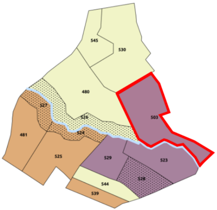


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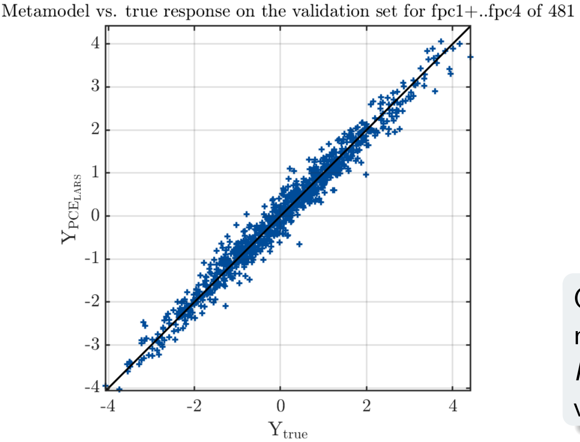
Figure – Sobol' indices for areal unit UH503. Bar colours refer to soil types.



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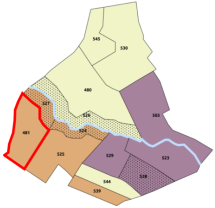

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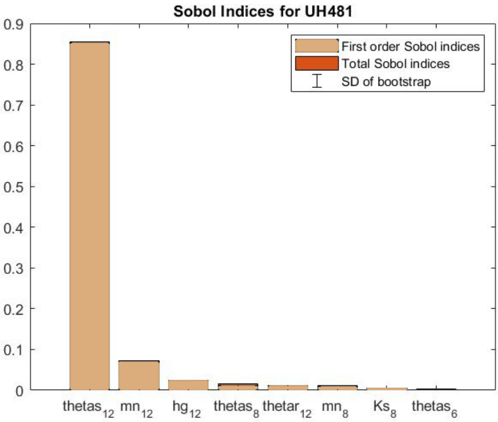
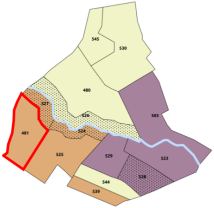


Figure – Sobol' indices for areal unit UH481. Bar colours refer to soil types.



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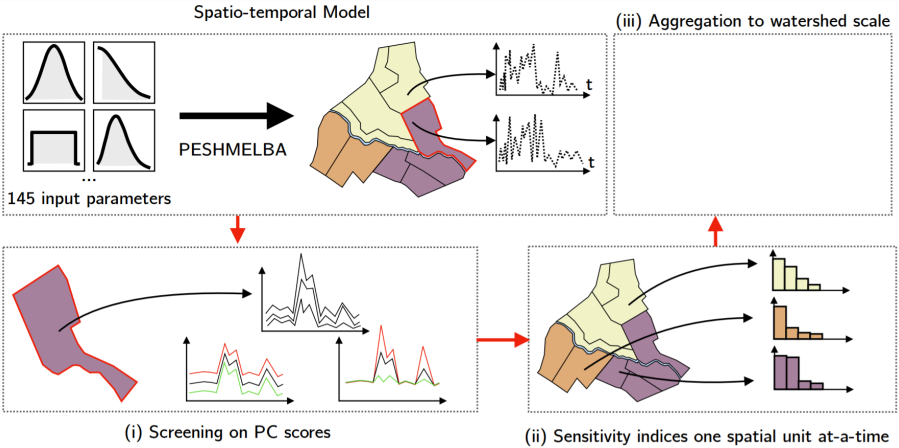
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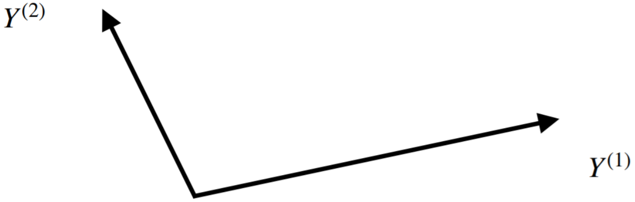
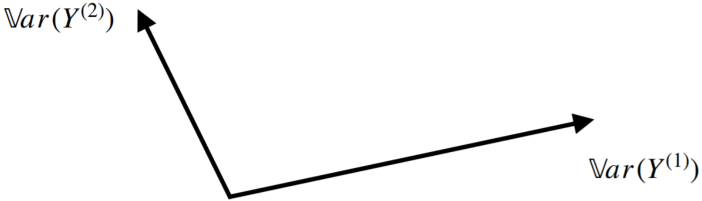
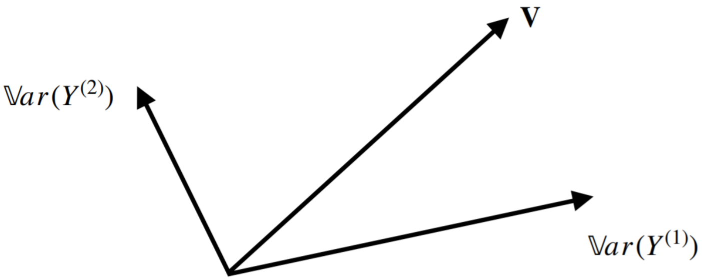
$Y^{(1)}$


$Y^{(2)}$
$Y^{(1)}$



$\mathbb{V}ar(Y^{(2)})$
$\mathbb{V}ar(Y^{(1)})$


$\mathbf{V}$
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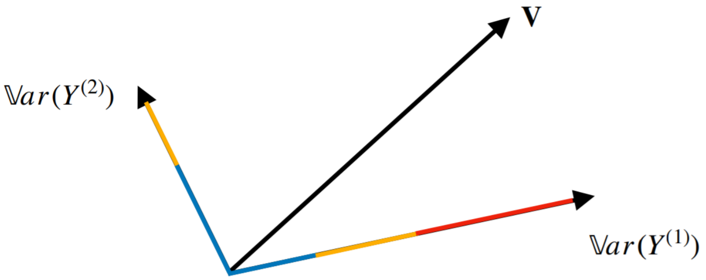
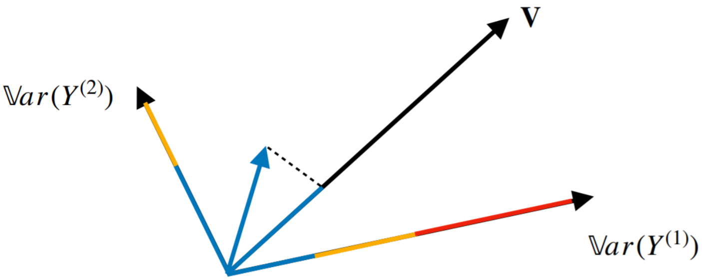


$$\mathbb{V}ar(Y^{(1)}) = \mathbb{V}ar(\mathbb{E}[Y^{(1)}|X_1]) + \mathbb{V}ar(\mathbb{E}[Y^{(1)}|X_2]) +$$

$$\mathbb{V}ar(\mathbb{E}[Y^{(1)}|X_1, X_2]) - \mathbb{V}ar(\mathbb{E}[Y^{(1)}|X_1]) - \mathbb{V}ar(\mathbb{E}[Y^{(1)}|X_2])$$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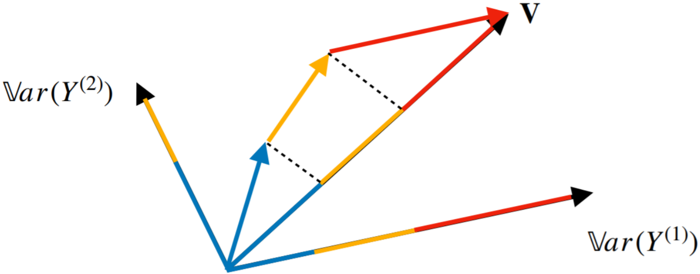
3. Spatial aspect

3.1. Vector projections *[Xu et al. 2019]*



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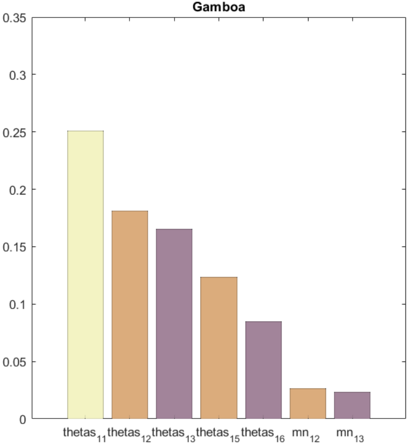
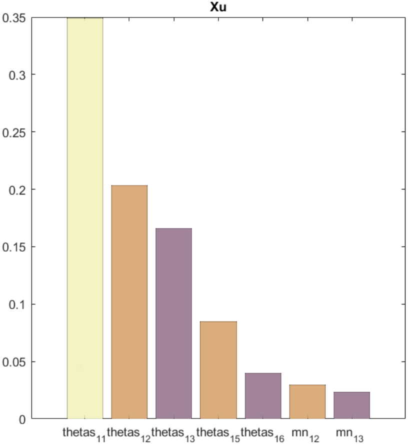
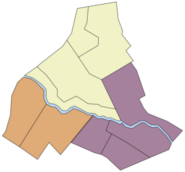
3.2. Gamboa vs. Xu, aggregation to the watershed scale

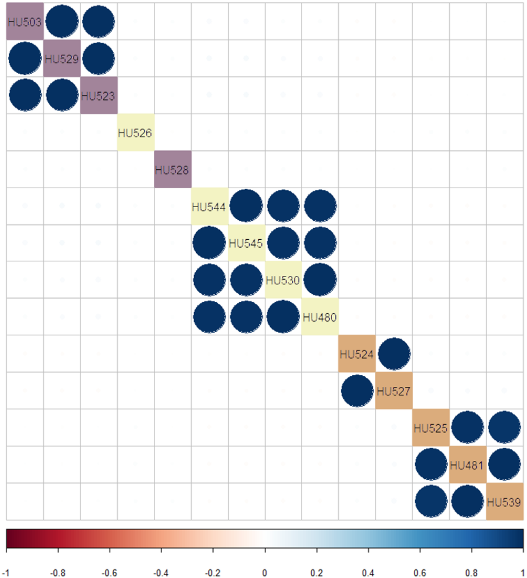


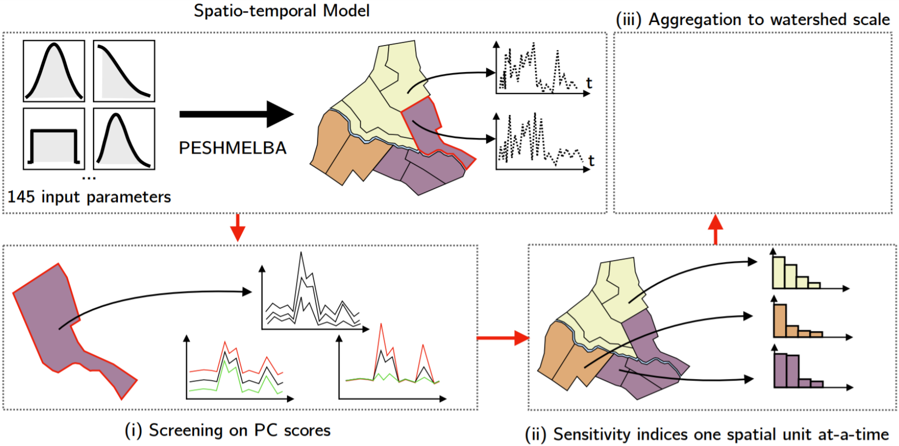


3. Spatial aspect

3.3. Correlations among outputs



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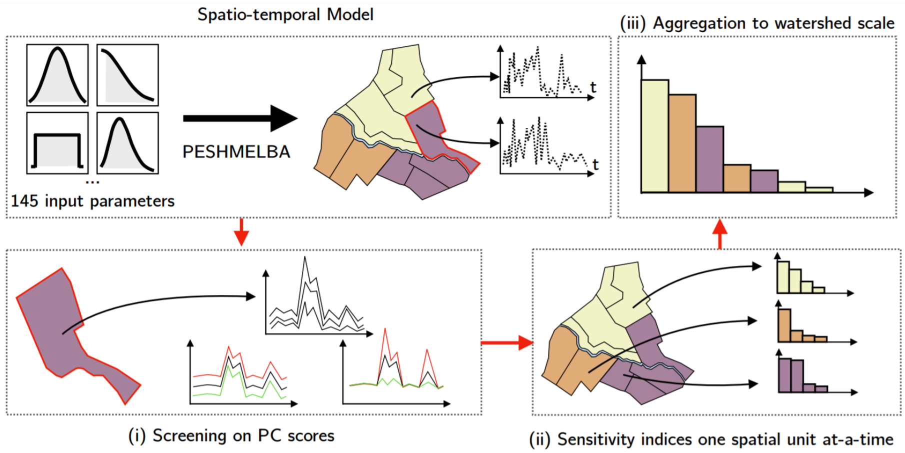
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  - Further adaptations : replace polynomial chaos expansion metamodel with more flexible metamodels (random forests).

Merci de votre attention !

Bibliography

- M. Lamboni, H. Monod, and D. Makowski. Multivariate sensitivity analysis to measure global contribution of input factors in dynamic models. Reliability Engineering System Safety, 96(4) :450–459, April 2011.
- E. Rouzies, C. Lauvernet, C. Barachet, T. Morel, F. Branger, I. Braud, and N. Carluer. From agricultural catchment to management scenarios : A modular tool to assess effects of landscape features on water and pesticide behavior. Science of The Total Environment, 671 :1144–1160, June 2019.
- L. Xu, Z. Lu, and S. Xiao. Generalized sensitivity indices based on vector projection for multivariate output. Applied Mathematical Modelling, 66 :592–610, February 2019.